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Production of Dirac Particles Due to Riccion Coupling

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It has been noted that at high energy the Ricci scalar is manifested in two different ways, as a matter field as well as a geometrical field (which is its usual nature even at low energy). Here, using the material aspect of the Ricci scalar, its interaction with Dirac spinors is considered in four-dimensional curved spacetime. We find that a large number of fermion–antifermion pairs can be produced by the exponential expansion of the early universe.

1. INTRODUCTION

Einstein's theory of gravity is successful at low energy (long distances), but, at high energy (short distances), this theory is problematic in two ways: (1) it is nonrenormalizable and (2) Einstein's field equations exhibit solutions having pointlike singularities (Hawking and Penrose, 1970), where physical laws collapse. So Einstein's theory needs a modification at high energies.

In this connection, there have been efforts to study higher derivative gravity (Utiyama and DeWitt, 1962; Stelle, 1977; Fradkin and Tseylin, 1982; Avramidy and Barvinsky, 1985; Parker and Toms, 1984; Buchbinder *et al.*, 1992; Srivastava and Sinha, 1993, 1994), which incorporates the principle of general covariance, the most basic principle of the general theory of relativity. Higher derivative gravity is obtained by adding higher order terms in R (the Ricci scalar) such as R^2, $R_{\mu\nu}R^{\mu\nu}$ ($R_{\mu\nu}$ are the components of the Ricci tensor), $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ ($R_{\mu\nu\rho\sigma}$ are the components of the Riemann curvature tensor), R^3, etc., to the Einstein–Hilbert Lagrangian which is linear in R. It

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is not always necessary to add terms like $R_{\mu\nu}R^{\mu\nu}$ and $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$. Some authors have taken a Lagrangian containing second- or higher order polynomials of R (Whitt, 1984; Barrow and Cotsakis, 1988, 1991). According to the definition of the Ricci scalar R, it contains the second-order derivative and the square of the first-order derivative of the components of the metric tensor with respect to space-time coordinates. The components of the metric tensor $g_{\mu\nu}$ are defined by

$$ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu \tag{1.1}$$

which shows that the $g_{\mu\nu}$ are dimensionless. Hence, on adopting natural units ($\hbar = c = k_B = 1$, where $\hbar$, c, and k_B have their usual meanings), one finds the dimension of R as $[\text{mass}]^2$. So, if the Lagrangian contains R^2-terms, coupling constants will remain dimensionless in 4-dimensional space-time. But if higher order terms of R (higher than R^2) are present in the Lagrangian, to maintain the dimensionless property of the action (the action has the same dimension as $\hbar$, which is dimensionless in natural units), either coupling constants acquire dimension or the dimension of space-time is higher than 4. In such cases, the theory is not renormalizable. But this does not mean that the theory will be uninteresting. Many interesting results can be derived even from a nonrenormalizable theory of gravity (Barrow and Cotsakis, 1988, 1991). Here we shall take the action for R^2-gravity to be

$$S_g = \int d^4x\, (-\tilde{g})^{1/2}\left(\frac{R}{16\pi G} + bR^2 + cR_{\mu\nu}R^{\mu\nu} + dR_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right) \tag{1.2}$$

where G is the gravitational constant of dimension $[\text{mass}]^{-2}$ (which is often taken equal to the Newtonian gravitational constant, $G_N \simeq M_P^{-2}$, where M_P is the Planck mass), the coupling constants b, c, and d are dimensionless, and $\tilde{g}$ is the determinant of $g_{\mu\nu}$.

It is important to mention here that higher derivative theories of gravity are not unitary within usual perturbation theory. In the perturbative approach, analysis of the theory at the quantum level starts with the expansion of components of the metric tensor $g_{\mu\nu}$ around the flat background space-time with components of the Minkowskian metric tensor $\eta_{\mu\nu}$. Later, modifying the theory by the Faddeev–Popov method (Buchbinder *et al.*, 1992), the kinetic energy matrix for $h_{\mu\nu} = (16\pi G)^{-1/2}(g_{\mu\nu} - \eta_{\mu\nu})$ is obtained which yields the Feynman propagator for $h_{\mu\nu}$. This tree-level propagator contains some terms corresponding physically to massive tachyonic ghosts. But this kind of problem is not expected in a nonperturbative approach.

In our earlier work (Srivastava and Sinha, 1993, 1994) as well as in the present paper, without expanding $g_{\mu\nu}$ around flat background space-time (without using a perturbative approach), we discuss the theory of higher derivative gravity. Moreover, the $g_{\mu\nu}$ are treated as classical fields. So, the problem of nonunitarity does not arise here. We have noted that at high energy the Ricci scalar R manifests itself in two different ways: (1) as a spinless matter field and (2) as a geometrical field (which is its usual role in gravitational theories), whereas it behaves only as a geometrical field at low energy. In quantum field theory, fields are treated as physical concepts describing elementary particles. So, particles described by the material aspect of the Ricci scalar are hereafter called riccions (which are new particles, different from gravitons, in the scenario of pure gravitational theories). Here, we obtain the Klein–Gordon equation for R in curved spaces, which is expected to provide a physically reasonable propagator like other scalar fields in curved spaces, without any ghost term breaking unitarity of the theory. In Section 2 we discuss the condition for riccions not to behave like tachyons. However, in the present paper the problem addressed is different. From dimensional considerations, the material aspect of R is represented by $\tilde{R} = \eta R$ (where η is a number of unit magnitude and the dimension of length, to have the mass dimension of $\tilde{R}$ equal to one, like other scalar fields in natural units). Thus, riccions are massive spinless particles, whereas gravitons are supposed to be massless spin-2 particles.

In the present paper, we are interested in the interaction of $\tilde{R}$ with Dirac spinor ψ. We consider the case when $\tilde{R}$ undergoes spontaneous symmetry breaking (under a temperature-dependent Higgs-like potential) (Srivastava and Sinha, 1993) leading to a phase transition from the state $\tilde{R} = 0$ to $|\tilde{R}| = \frac{1}{2}(T_c^2 - T^2)^{1/2}$, where T (T_c) is the temperature (critical temperature). The interaction term in the action for ψ is taken as $g\tilde{R}\bar{\psi}\psi$ (which is the Yukawa coupling of $\tilde{R}$ and ψ with g a constant). The term $\frac{1}{2}g\tilde{R}\bar{\psi}\psi$ also behaves like a mass term of ψ which vanishes at $T = T_c$, but acquires more and more mass as T falls below T_c. When $T << T_c$, the mass term for ψ is $\frac{1}{2}gT_c\bar{\psi}\psi$. The main focus in this paper lies in the study of the production of spin-$\frac{1}{2}$ particles as a result of a Yukawa coupling of spinors with $\tilde{R}$. It is found that a large number of Dirac particles are produced as a result of interaction of spinors with $\tilde{R}$ during the exponentially expanding phase of the early universe caused by spontaneous symmetry breaking when $\tilde{R}$ acquires the constant value $\frac{1}{2}T_c$ at $T << T_c$.

The paper is organized as follows. Section 2 contains a brief review of our earlier work (Srivastava and Sinha, 1993), where the dual role of Ricci scalar was discussed and the exponentially expanding model of the early universe was derived after spontaneous symmetry breaking. In Section 3, interaction of riccions and spin-$\frac{1}{2}$ fermions is introduced and the resulting Dirac equation is derived. In Section 4, the Dirac equation is solved. Section 5 is the concluding section, where the production of particles and its cosmological implications are discussed. Natural units are used throughout the paper.

2. DUAL ROLE OF RICCI SCALAR AT HIGH ENERGY

In earlier papers (Srivastava and Sinha, 1993, 1994), we showed, using the generalized Gauss–Bonnet theorem, which states that in a four-dimensional space-time

$$\int d^4x\,(-g)^{1/2}(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2)$$

is a topological invariant (Birrell and Davies, 1982) and is equal to χ (the Euler number), that the action for gravity (1.2) reduces to

$$S_g = \int d^4x\,(-\tilde{g})^{1/2}\left(\frac{R}{16\pi G} + \alpha R_{\mu\nu}R^{\mu\nu} + \beta R^2\right) + \chi \tag{2.1}$$

where α and β [being linear combinations of the coupling constants b, c, and d used in the action (1.2)] are dimensionless coupling constants.

Before going into further details, it is important to see the relative dominance between R and R^2 terms. In natural units, in terms of mass scale, $(16\pi G)^{-1}R$ corresponds to $[16\pi(G/G_0)]^{-1}M_P^2M^2$ and $(\alpha R_{\mu\nu}R^{\mu\nu} + \beta R^2)$ corresponds to $(\alpha + \beta)M^4$. So, it is found that when $M > [16\pi(\alpha + \beta)(G/G_0)]^{-1/2}M_P$, R^2 terms will dominate over the linear term $(16\pi G)^{-1}R$ and $(16\pi G)^{-1}R$ will dominate over R^2 terms when $M < [16\pi(\alpha + \beta)(G/G_0)]^{-1/2}M_P$. At energy mass scales $M << [16\pi(\alpha + \beta)(G/G_0)]^{-1/2}M_P$, R^2 terms will be almost insignificant compared to the Einstein–Hilbert term $(16\pi G)^{-1}R$. In other words, at sufficiently low energy, Einstein's theory of gravity will take over R^2 gravity. But, as discussed above, R^2 terms are very important at high energy and play a very important role in the theory of gravity. Thus the above modification to Einstein's theory of gravity is relevant at high energy only.

The invariance of S_g given by equation (2.1) yields, under the transformation $g^{\mu\nu} \to g^{\mu\nu} + \delta g^{\mu\nu}$, the field equations

$$\begin{aligned}&(16\pi G)^{-1}\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) + \alpha\left(R^{\alpha}_{\mu;\nu\alpha} - \Box R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\Box R\right.\\&\left.+ 2R^{\alpha}_{\mu}R_{\alpha\nu} - \frac{1}{2}g_{\mu\nu}R^{\gamma\delta}R_{\gamma\delta}\right) + \beta\left(2R_{;\mu\nu} - 2g_{\mu\nu}\Box R - \frac{1}{2}g_{\mu\nu}R^2\right.\\&\left.+ 2RR_{\mu\nu}\right) = 0\end{aligned} \tag{2.2}$$

where the semicolon denotes the covariant derivative and $\Box R = R^{;\mu}{}_{;\mu}$. Taking the trace of these field equations, we obtain the Klein–Gordon equation for

R in curved space-time, given as (Fradkin and Tseylin, 1982; Avramidy and Barvinsky, 1985; Srivastava and Sinha, 1993, 1994; Mayer and Schmidt, 1993)

$$(\Box + m^2)R = 0 \tag{2.3a}$$

where

$$m^2 = [8\pi G(5\alpha + 12\beta)]^{-1} \tag{2.3b}$$

It is interesting to observe from equations (2.3) that the Ricci scalar R behaves like a spinless matter field with mass given by (2.3b). Here m is real and nondivergent which is possible only when $(5\alpha + 12\beta) > 0$, leading to either of the three cases (1) $\alpha > 0$, $\beta > 0$, (2) $\alpha < 0$, $\beta > (5/12)|\alpha|$, and (3) $\alpha > (12/5)|\beta|$, $\beta < 0$. Thus, to make the theory free from tachyon ghosts, we can impose any one of three constraints on the dimensionless coupling constants α and β. Only on knowing explicit values of α and β can one decide which constraint will be suitable for the theory. Mayer and Schmid (1993) have called R a "spinless graviton." Here, as mentioned above, we call $\tilde{R} = \eta R$ a *riccion.*

Multiplying (2.3a) by η, we obtain

$$(\Box + m^2)\tilde{R} = 0 \tag{2.4a}$$

with m defined by (2.3b). Equation (2.4a) can also be derived from the action

$$S_{\tilde{R}} = \frac{1}{2}\int d^4x\,(-\tilde{g})^{1/2}(g^{\mu\nu}\,\partial_\mu\tilde{R}\,\partial_\nu\tilde{R} - m^2\tilde{R}^2) \tag{2.4b}$$

demanding its invariance with respect to the transformation $\tilde{R} \to \tilde{R} + \delta\tilde{R}$. In the action given by (2.4b), $\frac{1}{2}m^2\tilde{R}^2$ is the potential term. If we accept $\tilde{R}$ as a spinless matter field, in principle, we can also use a temperature-dependent Higgs-like potential for $\tilde{R}$ (like other scalar matter fields) as

$$V^T(\tilde{R}) = -\frac{1}{2}m^2\left(\tilde{R}^2 + \frac{T^2}{12}\right) + \frac{1}{4}\lambda\tilde{R}^4 + \frac{1}{8}\lambda T^2\tilde{R}^2 - \frac{\pi^2}{90}T^4 \tag{2.5}$$

in place of $\frac{1}{2}m^2\tilde{R}^2$ in (2.4b). In (2.5), λ is a dimensional coupling constant and T is the temperature. As a result, the invariance of the action

$$S'_{\tilde{R}} = \int d^4x\,(-\tilde{g})^{1/2}\left[\frac{1}{2}g^{\mu\nu}\,\partial_\mu\tilde{R}\,\partial_\nu\tilde{R} - V^T(\tilde{R})\right] \tag{2.6a}$$

under the transformation $\tilde{R} \to \tilde{R} + \delta\tilde{R}$ leads to

$$\Box\tilde{R} = -\frac{\partial V^T}{\partial\tilde{R}} = m^2\tilde{R} - \lambda\tilde{R}^3 - \frac{1}{4}\lambda T^2\tilde{R} \tag{2.6b}$$

Though $V^T(\tilde{R})$ can be written for $\tilde{R}$ after accepting it as a spinless matter field at high energy level, one may ask whether it is possible to write a higher derivative gravitational action leading to (2.6b) in the way in which S_g given by (2.1) leads to (2.3a). The answer to this question is provided by the action S'_g given as

$$S'_g = \int d^4x\,(-\tilde{g})^{1/2}\left[-\frac{R}{16\pi G} + \frac{1}{8}\lambda T^2(5\alpha + 12\beta)R + \alpha R_{\mu\nu}R^{\mu\nu} + \beta R^2 - \frac{\lambda\eta^2(5\alpha + 12\beta)}{2}R^3\right] \tag{2.6c}$$

We now demonstrate that the gravitational action S'_g given by (2.6c) leads to (2.6b). Invariance of S'_g given by (2.6c) under the transformation $g^{\mu\nu} \to g^{\mu\nu} + \delta g^{\mu\nu}$ yields the field equations

$$\begin{aligned}
&\left[\frac{1}{8}\lambda T^2(5\alpha + 12\beta) - (16\pi G)^{-1}\right]\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right)\\
&+ \alpha\left(R^{\alpha}_{\mu;\nu\alpha} - \Box R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\Box R + 2R^{\alpha}_{\mu}R_{\alpha\nu} - \frac{1}{2}g_{\mu\nu}R^{\gamma\delta}R_{\gamma\delta}\right)\\
&+ \beta\left(2R_{;\mu\nu} - 2g_{\mu\nu}\Box R - \frac{1}{2}g_{\mu\nu}R^2 + 2RR_{\mu\nu}\right)\\
&- \frac{\lambda\eta^2(5\alpha + 12\beta)}{2}\left(6R^2_{;\mu\nu} - 6g_{\mu\nu}\Box R^2 - \frac{1}{2}g_{\mu\nu}R^3 + 3R^2R_{\mu\nu}\right) = 0
\end{aligned}$$

On taking the trace of these equations, we find the equation

$$\Box\tilde{R} = \left(m^2 - \frac{1}{4}\lambda T^2\right)\tilde{R} - \lambda\tilde{R}^3 + 18\lambda\eta^3\Box\tilde{R}^2 \tag{2.6d}$$

As $\Box R^2$ is a total divergence, we have

$$\int d^4x\,(-\tilde{g})^{1/2}\Box R^2 = 0$$

which yields

$$g^{\mu\nu}\frac{\delta}{\delta g^{\mu\nu}}\int d^4x\,(-\tilde{g})^{1/2}\Box R^2 = 0$$

implying that

$$\Box\tilde{R}^2 = 0 \tag{2.6e}$$

Using (2.6e) in (2.6d), we obtain (2.6b) from the action (2.6c).

If the cosmological model of the early universe is spatially homogeneous and isotropic, it can be given by the line element

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2) \tag{2.7}$$

showing the flatness of t = const hypersurface.

The four components of the timelike vector u^μ are normalized as

$$u^\mu u_\mu = +1 \tag{2.8}$$

The components of energy-momentum tensor for $\tilde{R}$ can be derived from the action with a potential term V^T using the definition

$$T_{\mu\nu} = \frac{2}{(-\tilde{g})^{1/2}} \frac{\delta s_{\tilde{R}}}{\delta g^{\mu\nu}}$$

As a result, we obtain components of the energy-momentum tensor for $\tilde{R}$ as

$$\begin{aligned} T_{\mu\nu} = {} & \partial_\mu \tilde{R}\, \partial_\nu \tilde{R} - g_{\mu\nu}\left[\frac{1}{2}\,\partial^\sigma \tilde{R}\, \partial_\sigma \tilde{R} - V^T(\tilde{R})\right] \\ & - 2\eta R_{\mu\nu} \Box\tilde{R} + 2\eta m^2 \tilde{R} R_{\mu\nu} - 2\eta\lambda \tilde{R}^3 R_{\mu\nu} \\ & - \frac{1}{2}\,\eta\lambda T^2 \tilde{R} R_{\mu\nu} + 4\eta m^2 \tilde{R}_{;\mu\nu} - 4\eta m^2 g_{\mu\nu} \Box\tilde{R} \\ & - 4\eta\lambda \tilde{R}^3_{;\mu\nu} + 4\eta\lambda g_{\mu\nu} \Box R - \lambda T^2 \tilde{R}_{;\mu\nu} + \lambda T^2 g_{\mu\nu} \Box\tilde{R} \end{aligned} \tag{2.9}$$

In the case of other scalar fields ϕ, with Lagrangian density $\mathcal{L} = \frac{1}{2}(g^{\mu\nu}\, \partial_\mu\phi\, \partial_\nu\phi - V^T(\phi))$. $T^{(\phi)}_{\mu\nu} = \partial_\mu\phi\, \partial_\nu\phi - g_{\mu\nu}\mathcal{L}$. The $T_{\mu\nu}$ for $\tilde{R}$ given by (2.9) contains some other terms in addition to $\partial_\mu\tilde{R}\, \partial_\nu\tilde{R} - g_{\mu\nu}[\frac{1}{2}\partial^\sigma\tilde{R}\, \partial_\sigma\tilde{R} - V^T(\tilde{R})]$, due to the dependence of $\tilde{R}$ on $g_{\mu\nu}$.

Now, using equations (2.8) and (2.9), we obtain

$$\begin{aligned} u^\mu u^\nu T_{\mu\nu} = {} & u^\mu u^\nu\, \partial_\mu \tilde{R}\, \partial_\nu \tilde{R} - \frac{1}{2}\,\partial^\sigma \tilde{R}\, \partial_\sigma \tilde{R} + V^T(\tilde{R}) \\ & + u^\mu u^\nu \Big(-2\eta R_{\mu\nu} \Box\tilde{R} + 2\eta m^2 \tilde{R} R_{\mu\nu} - 2\eta\lambda \tilde{R}^3 R_{\mu\nu} \\ & - \frac{1}{2}\,\eta\lambda T^2 \tilde{R} R_{\mu\nu} + 4\eta m^2 \tilde{R}_{;\mu\nu} - 4\eta m^2 g_{\mu\nu} \Box\tilde{R} \\ & - 4\eta\lambda \tilde{R}^3_{;\mu\nu} + 4\eta\lambda g_{\mu\nu} \Box\tilde{R}^3 - \lambda T^2 \tilde{R}_{;\mu\nu} + \lambda T^2 g_{\mu\nu} \Box\tilde{R} \Big) \end{aligned} \tag{2.10}$$

where $u^\mu \equiv (1, 0, 0, 0)$ in comoving coordinates.

Taking the matter aspect of $\tilde{R}$, one can discuss spontaneous symmetry breaking. The vacuum state is given by

$$\frac{\partial V^T}{\partial \tilde{R}} = 0$$

or

$$-m^2\tilde{R} + \lambda\tilde{R}^3 + \frac{\lambda}{4}\tilde{R}T^2 = 0 \tag{2.11}$$

The turning points of V^T are given by

$$\tilde{R} = 0 \tag{2.12a}$$

$$|\tilde{R}| = \frac{1}{2}(T_c^2 - T^2)^{1/2} \tag{2.12b}$$

where

$$T_c = \frac{2m}{\sqrt{\lambda}} \tag{2.13}$$

Thus at $\tilde{R} = 0$, when $T \gtrsim T_c$, one finds from equation (2.10)

$$T_{\mu\nu}u^\mu u^\nu < 0 \tag{2.14}$$

which means that the energy condition is violated at temperature $T \gtrsim T_c$. Moreover, Einstein's theory has been modified. So, according to the Hawking–Penrose theorem (Hawking and Penrose, 1970), the cosmological model [given by the line element (2.7)] should be singularity-free. This implies that at $t = 0$, the cosmological model should bounce (Srivastava and Sinha, 1993).

In the model given by (2.7) the geometrical definition of $\tilde{R}$ yields

$$\tilde{R} = 6\eta\left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2\right] \tag{2.15}$$

where a dot denotes derivative with respect to time t.

Now, in the state $\tilde{R} = 0$, one obtains an ordinary differential equation

$$\frac{a''}{a} + \left(\frac{a'}{a}\right)^2 = 0 \tag{2.16}$$

where a prime denotes derivative with respect to t/t_P (t_P is the Planck time). The ordinary differential equation (2.16) integrates to

$$a^2 = a_0^2 + \frac{t}{t_P} \tag{2.17}$$

where $a(t = 0) = a_0 \neq 0$. Thus, in the state $\tilde{R} = 0$, the model of the early universe will expand as $(a_0^2 + t/t_P)^{1/2}$. As this expansion is adiabatic, entropy will remain conserved. So, the temperature will fall as $(a_0^2 + t/t_P)^{-1/2}$. When the temperature is sufficiently lower than T_c, $\tilde{R}$ will acquire a constant value $\tilde{R}_c$ such that

$$|\tilde{R}| = |\tilde{R}_c| = \frac{1}{2} T_c \tag{2.18}$$

In this state, we get the differential equation

$$\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 = \frac{T_c}{12\eta} \tag{2.19}$$

yielding the solution (Srivastava and Sinha, 1993)

$$a = a_c \sinh^{1/2}\left[(t - t_c)\left(\frac{T_c}{6\eta}\right)^{1/2} + 0.89\right] \tag{2.20}$$

where $a_c = (24/\eta T_c)^{1/4}$ and t_c is the time when $|\tilde{R}|$ acquires a constant value $|\tilde{R}_c| = \frac{1}{2}T_c$.

When $t > t_c - 0.89(6\eta/T_c)^{1/2}$, $a(t)$ asymptotically approaches

$$a \sim a_c \exp[(t - t_c)(T_c/24\eta)] \tag{2.21}$$

3. INTERACTION OF RICCIONS AND SPIN-$\frac{1}{2}$ FERMIONS

The manifestation of the dual nature of the Ricci scalar at high energy encourages one to study the effect of interaction of riccions with spin-$\frac{1}{2}$ particles, which form an important class of elementary particles. We take the complete action for the theory as

$$S = \frac{1}{2}\int d^4x\,(-\tilde{g})^{1/2}[g^{\mu\nu}\,\partial_\mu\tilde{R}\,\partial_\nu\tilde{R} - V^T(\tilde{R})]$$
$$+ \frac{1}{2}\int d^4x\,(-\tilde{g})^{1/2}[\bar{\psi}i\not{D}\psi - g\tilde{R}\bar{\psi}\psi + \text{complex conjugate}] \tag{3.1}$$

where $\bar{\psi} = \psi^\dagger\tilde{\gamma}^0$, $\not{D} = \gamma^\mu(\partial_\mu - \Gamma_\mu)$ (γ^μ are Dirac matrices in curved space-time) (Srivastava, 1989), and

$$\Gamma_\mu = -\frac{1}{4}(\partial_\mu h_a^\rho + \Gamma^\rho_{\sigma\mu}h_a^\sigma)g_{\nu\rho}h_b^\nu\tilde{\gamma}^b\tilde{\gamma}^a \tag{3.2}$$

In (3.2), $\Gamma^{\rho}_{\sigma\mu}$ are affine connections and $\tilde{\gamma}^a$ ($a = 0, 1, 2, 3$) are Dirac matrices in flat space-time satisfying the anticommutation rule

$$\{\tilde{\gamma}^a, \tilde{\gamma}^b\} = 2\eta^{ab} \tag{3.3}$$

where $\eta^{ab} \equiv \text{diag}(1, -1, -1, -1)$. The orthonormal tetrad components are defined as

$$h^{\mu}_a h^{\nu}_b g_{\mu\nu} = \eta_{ab} \tag{3.4}$$

With the help of the tetrad components h^{μ}_a, one can relate γ^{μ} and $\tilde{\gamma}^a$ as

$$\gamma^{\mu} = h^{\mu}_a \tilde{\gamma}^a \tag{3.5}$$

satisfying the anticommutation relation

$$\{\tilde{\gamma}^{\mu}, \tilde{\gamma}^{\nu}\} = 2g^{\mu\nu} \tag{3.6}$$

Imposing the condition

$$\frac{2}{(-\tilde{g})^{1/2}} \frac{\delta S}{\delta \bar{\psi}} = 0$$

we get the Dirac equation for ψ as

$$i\gamma^{\mu} D_{\mu}\psi - g\tilde{R}\psi = 0 \tag{3.7}$$

The geometry of the model under consideration is given by the line element (2.7), which is rewritten in terms of conformal time

$$\tau = \int^{t} \frac{dt'}{a(t')} \tag{3.8}$$

as

$$ds^2 = A^2(\tau)(d\tau^2 - dx^2 - dy^2 - dz^2) \tag{3.9}$$

As a result, the orthonormal tetrad components are

$$h^0_0 = h^1_1 = h^2_2 = h^3_3 = \frac{1}{A(\tau)} \tag{3.10}$$

Now the Dirac matrices γ^{μ} are

$$\gamma^{\mu} = \frac{1}{A(\tau)} \tilde{\gamma}^a \delta^{\mu}_a \tag{3.11}$$

and the Dirac equation (3.7) is rewritten as

$$[i(\tilde{\gamma}^0\partial_0 + \tilde{\gamma}^1\partial_1 + \tilde{\gamma}^2\partial_2 + \tilde{\gamma}^3\partial_3) - gA(\tau)\tilde{R}]\psi = 0 \tag{3.12a}$$

where

$$\Psi = A^{3/2}(\tau)\psi \tag{3.12b}$$

As mentioned above, in the introductory section, the interaction term behaves like a mass term. The mass acquired by ψ in the state $\tilde{R} = 0$ is zero, but has a definite value $\frac{1}{2}gT_c$ in the state $\tilde{R} = \tilde{R}_c$.

4. SOLUTION OF DIRAC EQUATION

For the purpose of second quantization, the general wave solution of the Dirac equation (3.12) can be written for discrete modes k and spin s as

$$\Psi = \sum_{s=\pm 1} \sum_{k} (b_{k,s}\Psi_{\mathrm{I}k,s} + d^{\dagger}_{-k,s}\Psi_{\mathrm{II}k,s}) \tag{4.1a}$$

$$\Psi^{\dagger} = \sum_{s=\pm 1} \sum_{k} (\overline{\Psi}_{\mathrm{I}k,s}\tilde{\gamma}_0 b^{\dagger}_{k,s} + \overline{\Psi}_{\mathrm{II}k,s}\tilde{\gamma}_0 d_{-k,s}) \tag{4.1b}$$

where

$$\Psi_{\mathrm{I}k,s} = f_{k,s}(\tau)e^{-ik_a x^a}u_s \tag{4.2a}$$

$$\Psi_{\mathrm{II}k,s} = g_{k,s}(\tau)e^{ik_a x^a}\hat{u}_s \tag{4.2b}$$

with

$$u_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \qquad u_{-1} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\hat{u}_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad \text{and} \quad \hat{u}_{-1} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{4.3}$$

In (4.1), $b^{\dagger}_{k,s}$ ($b_{k,s}$) are creation (annihilation) operators for positive-energy particle and $d_{-k,s}$ ($d^{\dagger}_{-k,s}$) are creation (annihilation) operators for negative-energy particles.

Now connecting equations (3.12) and (4.1), one gets the following equations for a particular mode k and spin s at temperature $T << T_c$:

$$\left(i\tilde{\gamma}^0\partial_0 + i\tilde{\gamma}^a\partial_a - \frac{1}{2}gAT_c\right)\Psi_{\mathrm{I}k,s} = 0 \tag{4.4a}$$

$$\left(i\tilde{\gamma}^0\partial_0 + i\tilde{\gamma}^a\partial_a - \frac{1}{2}gAT_c\right)\Psi_{\mathrm{II}k,s} = 0 \tag{4.4b}$$

and their complex conjugates.

Operating with $(-i\tilde{\gamma}^0\partial_0 - i\tilde{\gamma}^a\partial_a - \frac{1}{2}gAT_c)$ on equations (4.4) from the left and using equations (4.2) as well as

$$\tilde{\gamma}^0 u_s = \epsilon u_s, \qquad \tilde{\gamma}^0 \hat{u}_s = \epsilon \hat{u}_s, \qquad \epsilon = \pm 1 \tag{4.5}$$

one gets

$$\frac{d^2 f_{k,s}}{d\tau^2} + \left(k^2 + \frac{i\epsilon g}{2} T_c\, \partial_0 A + \frac{g^2 T_c^2}{4} A^2\right) f_{k,s} = 0 \tag{4.6a}$$

and

$$\frac{d^2 g_{k,s}}{d\tau^2} + \left(k^2 + \frac{i\epsilon g}{2} T_c\, \partial_0 A + \frac{g^2 T_c^2}{4} A^2\right) g_{k,s} = 0 \tag{4.6b}$$

As $g_{k,s} = f_{-k,s}$, we shall concentrate on equation (4.6a). It has been discussed in Section 2 that the cosmological model expands according to the law given by (2.21) in the state $|\tilde{R}| = \frac{1}{2}T_c$ when $t > t_c - 0.89$.

Now, using equations (2.21) and (3.8), one gets

$$A(\tau) = -\frac{1}{H\tau} \tag{4.7a}$$

with

$$H^2 = \frac{T_c}{24\eta} \tag{4.7b}$$

and

$$\tau = -\frac{1}{Ha_c}\, e^{-H(t-t_c)} \tag{4.7c}$$

Now, with $A(\tau)$ given by equation (4.7a), the differential equation (4.6a) is rewritten as

$$\frac{d^2}{d\tau^2} f_{k,s} + \left(k^2 + \frac{i\epsilon g T_c}{2H\tau^2} + \frac{g^2 T_c^2}{4H^2\tau^2}\right) f_{k,s} = 0 \tag{4.8}$$

From equation (4.7c) it is obvious that

$$-\infty < \tau < 0 \tag{4.9}$$

corresponding to $-\infty < t < \infty$.

In the limit $\tau \to -\infty$, equation (4.8) reduces to

$$\frac{d^2}{d\tau^2} f_{k,s} + k^2 f_{k,s} = 0$$

which yields positive- and negative-energy normalized solutions $(2k)^{-1/2}e^{-ik\tau}$ and $(2k)^{-1/2}e^{ik\tau}$, respectively.

The ordinary differential equation (4.8) yields the general solutions

$$f_{k,+1} = \tau^{\nu} e^{\pm ik\tau} \, {}_1F_1(\nu; 2\nu; \mp 2ik\tau) \tag{4.10a}$$

and

$$f_{k,-1} = \tau^{\nu} e^{\pm ik\tau} (\mp 2ik\tau)^{1-2\nu} \, {}_1F_1(1 - \nu; 2 - 2\nu; \mp 2ik\tau) \tag{4.10b}$$

where

$$\nu = \frac{1}{2}\left\{1 \pm \left[1 - 4\left(\frac{g^2T_c^2}{4H^2} + \frac{i\epsilon g T_c}{2H}\right)\right]^{1/2}\right\}$$

$$\simeq \frac{1}{2}\left[1 \pm \left(\frac{gT_c}{H} + i\right)\right] \tag{4.10c}$$

(as $gT_c >> H$) and ${}_1F_1(a; b; x)$ is the confluent hypergeometric function.

We know a useful identity connecting the Bessel function and the confluent hypergeometric function (Whittaker and Watson, 1969)

$$J_l(z) = \frac{(z/2)^l}{(1 + l)^{1/2}} e^{\pm iz} \, {}_1F_1\left(\frac{1}{2} + l; 1 + 2l; \mp 2iz\right) \tag{4.11}$$

Using the identity (4.11), we can rewrite the solutions (4.10) as

$$f_{k,+1} = \tau^{1/2}\left(\frac{k}{2}\right)^{\mp(gT_c/2H)+i/2)}\left[1 \pm \left(\frac{gT_c}{2H} + \frac{i}{2}\right)\right]^{1/2} J_{\pm(gT_c/2H+i/2)}(k\tau) \tag{4.12a}$$

and

$$f_{k,-1} = \tau^{1/2}(\mp 2i)^{\mp(gT_c/H+i)} k^{\mp(gT_c/2H+i/2)} 2^{\pm(gT_c/2H+i/2)}\left[1 \mp \left(\frac{gT_c}{2H} + \frac{i}{2}\right)\right]^{1/2}$$

$$\times J_{\mp(gT_c/2H+i/2)}(k\tau) \tag{4.12b}$$

The normalized forms of equations (4.12) are

$$f_{k,+1} = \left(\frac{\pi}{4}\right)^{1/2} e^{(\pm 1 - i)\pi/4} e^{\mp i\pi g T_c/4H} \tau^{1/2} J_{\pm(gT_c/2H+i/2)}(k\tau) \tag{4.13a}$$

and

$$f_{k,-1} = \left(\frac{\pi}{4}\right)^{1/2} e^{(\mp 1 + i)\pi/4} e^{\pm i\pi g T_c/4H} \tau^{1/2} J_{\mp(gT_c/2H+i/2)}(k\tau) \tag{4.13b}$$

Using the asymptotic form of the Bessel function (Pipes and Harvill, 1970)

$$J_l(z) \xrightarrow[z\to\infty]{} \left(\frac{\pi z}{2}\right)^{-1/2} \cos\left(z - \frac{\pi l}{2} - \frac{\pi}{4}\right)$$

one can easily see that

$$\lim_{\tau\to-\infty} f_{k,+1} = \begin{cases} (2k)^{-1/2}e^{-ik\tau} & \text{for} \quad k > 0 \\ (2k)^{-1/2}e^{ik\tau} & \text{for} \quad k < 0 \end{cases}$$

We get the same result for $f_{k,-1}$. This signifies that the normalization $f_{k,s}(k\tau)$ has been done correctly for $\tau \to -\infty$.

Moreover, we also find that the Wronskian of equation (4.8) is constant. This means that the above normalization of the solution is correct for all τ in the interval mentioned. Thus equations (4.13) are normalized solutions for $-\infty < \tau \leq 0$.

Now, one can also write

$$g_{k,+1} = f_{-k,+1} = \left(\frac{\pi}{4}\right)^{1/2} e^{(\pm 1 - i)\pi/4} e^{\mp i\pi g T_c/4H} \tau^{1/2} J_{\pm(gT_c/2H + i/2)}(-k\tau) \tag{4.14a}$$

and

$$g_{k,-1} = f_{-k,-1} = \left(\frac{\pi}{4}\right)^{1/2} e^{(\mp 1 + i)\pi/4} e^{\pm i\pi g T_c/4H} \tau^{1/2} J_{\mp(gT_c/2H + i/2)}(-k\tau) \tag{4.14b}$$

For small values of z,

$$J_l(z) \simeq \frac{(z/2)^l}{(1 + l)^{1/2}} \tag{4.15}$$

For the purpose of analyzing the asymptotics of the solutions as $t \to \pm\infty$, the solutions given by (4.13) and (4.14) are written as

$$f_{k,+1} = \begin{cases} A_1 \tau^{1/2} J_{1/2(gT_c/H + i)}(k\tau) & \text{as} \quad t \to \infty \\ A_1 \tau^{1/2} J_{-1/2(gT_c/H + i)}(k\tau) & \text{as} \quad t \to -\infty \end{cases} \tag{4.16a}$$

$$f_{k,-1} = \begin{cases} A_2 \tau^{1/2} J_{1/2(gT_c/H + i)}(k\tau) & \text{as} \quad t \to \infty \\ A_2 \tau^{1/2} J_{-1/2(gT_c/H + i)}(k\tau) & \text{as} \quad t \to -\infty \end{cases} \tag{4.16b}$$

$$g_{k,+1} = \begin{cases} A_1 \tau^{1/2} J_{1/2(gT_c/H + i)}(-k\tau) & \text{as} \quad t \to \infty \\ A_1 \tau^{1/2} J_{-1/2(gT_c/H + i)}(-k\tau) & \text{as} \quad t \to -\infty \end{cases} \tag{4.16c}$$

$$g_{k,-1} = \begin{cases} A_2\tau^{1/2}J_{1/2(gT_c/H+i)}(-k\tau) & \text{as} \quad t \to \infty \\ A_2\tau^{1/2}J_{-1/2(gT_c/H+i)}(-k\tau) & \text{as} \quad t \to -\infty \end{cases} \tag{4.16d}$$

In equations (4.16)

$$A_1 = \left(\frac{\pi}{4}\right)^{1/2} e^{(\pm 1 - i)\pi/4} e^{\mp i\pi gT_c/4H} \tag{4.17a}$$

$$A_2 = \left(\frac{\pi}{4}\right)^{1/2} e^{(\mp 1 + i)\pi/4} e^{\pm i\pi gT_c/4H} \tag{4.17b}$$

and

$$\tau = -(24\eta^3/T_c)^{1/4} e^{-H(t-t_c)} \tag{4.17c}$$

Using the normalization condition for Ψ as (Birrell and Davies, 1982)

$$(\overline{\Psi}_{k,s}, \Psi_{k',s'}) = \int_{t=\text{const}} d^3x\, \overline{\Psi}_{k,s}\tilde{\gamma}^0\Psi_{k',s'} = \delta_{ss'}\delta_{kk'}$$

one can normalize $\Psi_{\mathrm{I}k,s}$ and $\Psi_{\mathrm{II}k,s}$ as

$$\Psi_{\mathrm{I}k,s} = \frac{i}{V^{3/2}}\left(\frac{T_c}{24\eta^3}\right)^{1/8} f_{k,s}e^{-ik_a x^a}u_s \tag{4.18a}$$

$$\Psi_{\mathrm{II}k,s} = \frac{i}{V^{3/2}}\left(\frac{T_c}{24\eta^3}\right)^{1/8} g_{k,s}e^{ik_a x^a}\hat{u}_s \tag{4.18b}$$

where V is the volume of 3-dimensional space, and $f_{k,s}$ and $g_{k,s}$ are given by equations (4.16).

5. PRODUCTION OF PARTICLES

The question of the production of spin-$\frac{1}{2}$ particles in radiation-dominated models of the universe has been addressed (Parker, 1971, 1977; Audretsch and Schafer, 1978) considering free spinor fields. Here we are interested in the production of Dirac particles in the exponentially expanding cosmological model (caused by the process of spontaneous symmetry breaking with temperature-dependent Higgs-like potential for the scalar field $\tilde{R}$) as a consequence of the interaction of spinors with riccions.

To study the spectrum of created spin-$\frac{1}{2}$ particles, we go over to quantum field theory in the Fock space formulation. The in-vacuum state (as $t \to -\infty$) $|0\rangle_{\text{in}}$ is defined as

$$b^{\text{in}}_{k,s}|0\rangle_{\text{in}} = d^{\text{in}}_{-k,s}|0\rangle_{\text{in}} = 0 \tag{5.1a}$$

$$\langle 0|0\rangle_{\text{in}} = 1 \tag{5.1b}$$

The out-vacuum state (at $t \to +\infty$) $|0\rangle_{\text{out}}$ is defined as

$$b^{\text{out}}_{k,s}|0\rangle_{\text{out}} = d^{\text{out}}_{-k,s}|0\rangle_{\text{out}} = 0 \tag{5.2a}$$

$$\langle 0|0\rangle_{\text{out}} = 1 \tag{5.2b}$$

In the in-region as well as the out-region the decomposed form of Ψ can be written as

$$\begin{aligned}\Psi &= \sum_{s=\pm 1} \sum_{k} [b^{\text{in}}_{k,s}\Psi^{\text{in}}_{\text{I}k,s} + (d^{\text{in}}_{-k,-s})^{\dagger}\Psi^{\text{in}}_{\text{II}(-k,-s)}] \\ &= \sum_{s=\pm 1} \sum_{k} [b^{\text{out}}_{k,s}\Psi^{\text{out}}_{\text{I}k,s} + (d^{\text{out}}_{-k,-s})^{\dagger}\Psi^{\text{out}}_{\text{II}(-k,-s)}]\end{aligned} \tag{5.3a}$$

$$\begin{aligned}\Psi^{\dagger} &= \sum_{s=\pm 1} \sum_{k} [(\Psi^{\text{in}}_{\text{I}k,s})^{\dagger}(b^{\text{in}}_{k,s})^{\dagger} + (\Psi^{\text{in}}_{\text{II}k,s})^{\dagger}d^{\text{in}}_{-k,-s}] \\ &= \sum_{s=\pm 1} \sum_{k} [(\Psi^{\text{out}}_{\text{I}k,s})^{\dagger}(b^{\text{out}}_{k,s})^{\dagger} + (\Psi^{\text{out}}_{\text{II}k,s})^{\dagger}d^{\text{out}}_{-k,-s}]\end{aligned} \tag{5.3b}$$

The Bogoliubov transformations for Fermi fields are (DeWitt, 1975)

$$b^{\text{out}}_{k,s} = b^{\text{in}}_{k,s}\alpha_{k,s} + d^{\text{in}}_{-k,-s}\beta_{k,s} \tag{5.4a}$$

$$(b^{\text{out}}_{k,s})^{\dagger} = \alpha^{*}_{k,s}(b^{\text{in}}_{k,s})^{\dagger} + \beta^{*}_{k,s}(d^{\text{in}}_{-k,-s})^{\dagger} \tag{5.4b}$$

$$(d^{\text{out}}_{-k,-s})^{\dagger} = b^{\text{in}}_{k,s}\alpha_{k,s} + (d^{\text{in}}_{-k,-s})^{\dagger}\beta_{k,s} \tag{5.4c}$$

$$d^{\text{out}}_{-k,-s} = \alpha^{*}_{k,s}(b^{\text{in}}_{k,s})^{\dagger} + \beta^{*}_{k,s}d^{\text{in}}_{-k,-s} \tag{5.4d}$$

The Bogoliubov coefficients $\alpha_{k,s}$ and $\beta_{k,s}$ satisfy the condition

$$|\alpha_{k,s}|^2 + |\beta_{k,s}|^2 = 1 \tag{5.5}$$

with

$$\alpha_{k,s} = \int_{t=t_1(\text{const})} d^3x\ \Psi^{\text{in}}_{\text{I}k,s}\psi^{\text{out}\dagger}_{\text{I}k,s} \tag{5.6a}$$

and

$$\beta_{k,s} = \int_{t=t_1(\text{const})} d^3x\ \Psi^{\text{in}}_{\text{II}(-k,-s)}\psi^{\text{out}\dagger}_{\text{II}(-k,-s)} \tag{5.6b}$$

as anticommutation relations, complete relations, and orthogonality conditions are defined as the t = const hypersurface.

Using equations (4.2), (4.16), and (4.17), we obtain

$$\alpha_{k,s} = \frac{(\pi/4)e^{+s\pi/2}[-(24/\eta T_c)^{1/4}e^{-H(t_1-t_c)}]^{1-i}}{[1 - \frac{1}{2}(gT_c/H + i)]^{1/2}[1 + \frac{1}{2}(gT_c/H + i)]^{1/2}}\left(\frac{k}{2}\right)^{-i} \tag{5.7a}$$

$$\beta_{k,s} = \frac{(\pi/4)e^{-s\pi/2}[-(24/\eta T_c)^{1/4}e^{-H(t_1-t_c)}]^{1-i}}{[1 - \frac{1}{2}(gT_c/H + i)]^{1/2}[1 + \frac{1}{2}(gT_c/H - i)]^{1/2}}\left(-\frac{k}{2}\right)^{-i} \tag{5.7b}$$

Equations (5.7) yield

$$|\alpha_{k,s}|^2 = \frac{e^{s\pi}(24/\eta T_c)^{1/2}e^{-2H(t_1-t_c)}}{8(1 + 24\eta g^2 T_c)}\{11.689 - \cos[\pi g(24\eta)^{1/2}T_c]\} \tag{5.8}$$

and

$$|\beta_{k,s}|^2 = \frac{e^{-s\pi}(24/\eta T_c)^{1/2}e^{-2H(t_1-t_c)}}{8(1 + 24\eta g^2 T_c)}\{11.689 - \cos[\pi g(24\eta)^{1/2}T_c]\} \tag{5.9}$$

All the above analysis in this section is based on the approximation of the Bessel function (for small arguments) given by equation (4.15). So, modes k are restricted by the condition

$$k^2 << (T_c/24\eta^3)^{1/2}\exp[2(t_1 - t_c)(T_c/24\eta)^{1/2}] \tag{5.10}$$

As a result, $|\beta_{k,s}|^2$ given by (5.9) is independent of k, so it is true for all k subject to the inequality (5.10).

Now using (5.5), one can compute the time t_1 as

$$t_1 = t_c + \frac{1}{2H}\ln\left(\frac{\cosh s\pi}{4(1 + 24\eta g^2 T_c)}\left(\frac{24}{\eta T_c}\right)^{1/2} \times \{11.689 - \cos[\pi g(24\eta)^{1/2}T_c)]\}\right) \tag{5.11}$$

This is a very good and important result, which says that only at t_1 given by equation (5.11) is equation (5.5) satisfied by $\alpha_{k,s}$ and $\beta_{k,s}$. Physically, this means that the production of spin-$\frac{1}{2}$ fermions (Dirac particles) is possible only at time t_1 explicitly defined by equation (5.11).

Since $t_1 > t_c$, equation (5.11) yields one condition on T_c as

$$4(\eta T_c/24)^{1/2}(1 + 24\eta g^2 T_c) > 12.689 \cosh \pi \tag{5.12}$$

Using equation (2.13) in the inequality (5.12), one gets

$$\frac{96}{\sqrt{3}} m^{3/2} > 12.689 \cosh \pi\left(\frac{\lambda^{3/4}}{g^2}\right)\eta^{3/2} \tag{5.13}$$

where m is the mass of the Ricci field. If G is taken equal to the Newtonian gravitational constant, which is M_p^{-2} (in natural units), one finds the condition on the coupling constants α, β, λ, and g as

$$g^2 < 10^{-27}[\lambda(5\alpha + 12\beta)]^{3/4} \tag{5.14}$$

The absolute probability of creation of no particles in a mode is given by (Parker, 1977; Mottola, 1985)

$$\begin{aligned} |{}_{\text{out}}\langle 0|0\rangle_{\text{in}}|^2 &= \prod_{k,s} |\alpha_{k,s}|^{-2} \\ &= \prod_k |\alpha_{k,+1}|^2 |\alpha_{k,-1}|^2 \\ &= \exp(\sum_k \ln|\alpha_{k,+1}|^{-2}|\alpha_{k,-1}|^{-2}) \\ &= \exp[2\zeta(0) \ln|\alpha_{k,+1}|^{-2}|\alpha_{k,-1}|^{-2}] \\ &= \exp(-\ln|\alpha_{k,+1}|^{-2}|\alpha_{k,-1}|^{-2}) \\ &= |\alpha_{k,+1}|^2|\alpha_{k,-1}|^2 \\ &= \frac{1}{4\cosh^2 \pi} \end{aligned} \tag{5.15}$$

which is true for all modes constrained by the condition (5.10).

Here we have used equations (5.8a) and (5.11), and $\zeta(0)$ is the Riemann zeta function, $\zeta(\nu)$ at $\nu = 0$, which is divergent but evaluated equal to $-\frac{1}{2}$ through analytic continuation.

The decay of the $|0\rangle_{\text{in}}$ state per unit volume is given by

$$\begin{aligned} \Gamma &= -V_4^{-1} \ln|{}_{\text{out}}\langle 0|0\rangle_{\text{in}}|^2 \\ &= \frac{3}{2\pi}\left(\frac{\eta T_c}{24}\right)^{5/4} \frac{\ln 4\cosh^2 \pi}{\exp[3(t_1 - t_c)T_c^{1/2}/24\eta] - 1} \end{aligned} \tag{5.16}$$

$$\begin{aligned} &= [(12/\pi)(T_c/24\eta)^{5/4} \ln(4\cosh^2 \pi)(1 + 24\eta g^2 T_c)^{3/2}] \\ &\times [\{\cosh \pi(24\eta/T_c)^{1/2}[11.689 - \cos \pi g(24\eta T_c)^{1/2}]\}^{3/2}] \\ &\quad - 8(1 + 24\eta g^2 T_c)^{3/2}]^{-1} \end{aligned} \tag{5.17}$$

Using the inequality (5.12), one can find that

$$\Gamma \simeq \frac{(T_c\eta)^2(1 + 24\eta g^2 T_c)^{3/2} \ln(4 \cosh^2 \pi)}{48\pi \cosh \pi[11.689 - \cos \pi g(24\eta T_c)^{1/2}]} \tag{5.18}$$

The in-vacuum state will decay, when particles will be created. So, decay per unit volume of the in-vacuum state is equivalent to the creation of particles per unit volume. Thus, the equality (5.18) shows that the creation of particles will be very high for modes of magnitude

$$|k| << \frac{\cosh^{1/2} \pi\{11.689 - \cos[\pi g(24\eta T_c)]^{1/2}\}^{1/2}}{2(1 + 24\eta g^2 T_c)^{1/2}} \tag{5.19}$$

which can be obtained by using inequality (5.10) and equation (5.11). Using inequality (5.14) and the definition of T_c given by equation (2.13), we can modify the inequality as

$$|k| << 4 \text{ GeV} \tag{5.20}$$

(here the gravitational constant has been taken equal to M_p^{-2}, the Newtonian gravitational constant). This result is very interesting, as it says that particles of mass <<4 GeV will be created at time $t_1 > t_c$. Observed elementary spin-$\frac{1}{2}$ particles have mass $\leq$1 GeV. So, the result (5.20) is consistent with observations.

Using the approximate form of the Bessel function for large arguments, which is given as

$$J_l(z) \simeq (\pi z/2)^{-1/2} \cos\left(2 - l\frac{\pi}{2} - \frac{\pi}{4}\right)$$

one can easily find that the probability of creation of particles is almost negligible in high modes for which

$$|k| \geq \frac{\cosh^{1/2} \pi}{2(1 + 24\eta g^2 T_c)^{1/2}} [11.689 - \cos \pi g(24\eta T_c)^{1/2}]^{1/2} \tag{5.21}$$

Thus, on the basis of the above analysis, we find that a large amount of energy produced due to the inflationary phase of the model will flow in the out-region, which may increase the entropy of the later universe.

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